Summary – Chapter 4 Functions

Every set of ordered pairs is a *relation*.
A *function* is a special type of relation in which every input has exactly one unique output.
Graphically, a function must pass the *vertical line test*.

Domain is the set of all inputs (independent variables – often represented by "x")
Range is the set of all outputs (dependent variables – often represented by y or f(x))
Zeros are the points on the function that make the output = 0. They are the x-intercepts.

A piecewise function is a relationship between two variables. They are a compilation of different functions defined over non-overlapping domains.

Transformations of Graphs

Shifts For c>0, *to obtain the graph of:*		
f(x)+c	*shift the graph of f(x)*	**upward** c units
f(x)-c	*shift the graph of f(x)*	**downward** c units
f(x+c)	*shift the graph of f(x)*	**left** c units
f(x-c)	*shift the graph of f(x)*	**right** c units

Stretches and compressions For c>1, *to obtain the graph of:*		
cf(x)	***stretch*** *the graph of f(x)*	**vertically** by a factor of c
(1/c)f(x)	***compress*** *the graph of f(x)*	**vertically** by a factor of c
f(cx)	***compress*** *the graph of f(x)*	**horizontally** by a factor of c
f(x/c)	***stretch*** *the graph of f(x)*	**horizontally** by a factor of c

Reflections *To obtain the graph of:*		
-f(x)	*reflect the graph of f(x)*	**about the x-axis**
f(-x)	*reflect the graph of f(x)*	**about the y-axis**

Remember if the transformation is outside the function, the result is exactly what is seen. The y-value is changed by exactly that amount. If the transformation involves "x" the graph will transform opposite of what you see. For example f(x+2) would move your graph two units to the left. (x– (–2)).

To see the impact of each type of transformation, on your graphing calculator or using desmos, graph the following and note how each changes in relation to the parent function.
F(x) = |x| g(x) = |x -2| h(x) = |x| + 2   k(x) = 2|x| m(x) = |4x|

All functions can be classified as even, odd or neither.

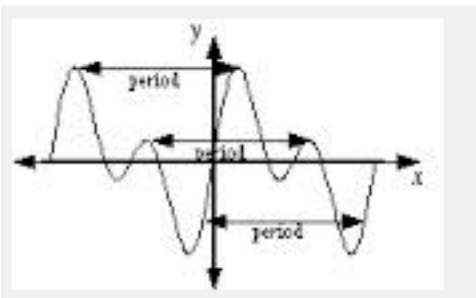
Even functions are symmetric over the y-axis f(x) = f(–x)
Odd functions are 180° point symmetric over (0,0)   f(–x) = –f(x)

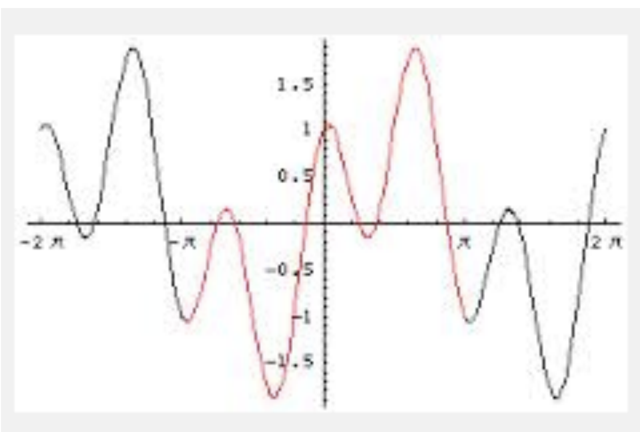
Some functions may have point or line symmetry and not be classified as even or odd.

*Line symmetry f(x) = ax2+bx + c is a quadratic function with line symmetry through the vertex . The equation of the line is x = –b/2a

*Point symmetry f(x) = ax3+bx2+cx+d is a cubic function with point symmetry around the point with x value –b/3a. To find the corresponding y-value of the point, substitute the x value into the original equation.

A periodic function is a function that repeats itself. The smallest interval required for a function to repeat is called the fundamental period. To find the fundamental period, look closely at the graph. The fundamental period can be measured as the horizontal distance between consecutive minimums or consecutive maximums or the amount of time for the pattern to repeat itself once.





In the graph above the fundamental period is 2π.

To evaluate the value of a periodic function that extends beyond the graph, divide the value by the period. The remainder should be used to find the corresponding output.
For example if the graph above is f(x), find f(13π).   Divide 4π/2π. The remainder is 0. Therefore f(4π) = f(0) = 1.

A function has an inverse. The inverse does not necessarily need to be a function. If it is the original function is called a one-to-one function and the graph of the original function will pass both the horizontal and vertical line test.

Finding the Inverse of a Function

Given the function $f(x)$ we want to find the inverse function, $f^{-1}(x)$.

1. First, replace $f(x)$ with y. This is done to make the rest of the process easier.
2. Replace every x with a y and replace every y with an x.
3. Solve the equation from Step 2 for y. This is the step where mistakes are most often made so be careful with this step.
4. Replace y with $f^{-1}(x)$. In other words, we've managed to find the inverse at this point!
5. Verify your work by checking that $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$ are both true. This work can sometimes be messy making it easy to make mistakes so again be careful.

http://tutorial.math.lamar.edu/Classes/CalcI/InverseFunctions.aspx