Name: ______________________________

623 Exponents and Logarithms

1. A radioactive substance decays in such a way that the amount of mass remaining after t days is given by the function $m(t) = 13e^{-0.015t}$ where m(t) is measured in kilograms.

   a.) Find the mass at t=0

   13

   b.) How much mass remains after 5 days?

   $m(5) = 13e^{-.015(5)}$

   $= 12.06$ g

2. The amount of time it takes to double the amount of an investment at an interest rate r compounded continuously is given by the formula $t = \frac{\ln 2}{r}$. Find the time required to double an investment at 6%, 2% and 19%.

   $\frac{\ln 2}{.06} = 11.55$ $\frac{\ln 2}{.02} = 34.65$ $\frac{\ln 19}{.02} = 3.68$

3. Solve each equation.

   a.) $\log_2 x = 5$

   $2^5 = x$

   $x = 32$

   b.) $\log_4 2 = x$

   $4^x = 2$

   $x \ln 4 = \ln 2$

   $x = .5$

   c.) $\ln x = 10$

   $e^{10} = x$

   d.) $3^{2x-1} = 5$

   $(2x-1)\ln 3 = \ln 5$

   $x = 1.23$

   e.) $4 + 3^{5x} = 8$

   $3^{5x} = 4$

   $5x \ln 3 = \ln 4$

   $x = .25$

   f.) $e^{2x+1} = 200$

   $2x + 1 = \ln 200$

   $x = 2.149$

   g.) $100(1.04)^{2t} = 300$

   $1.04^{2t} = 3$

   $2t \ln 1.04 = \ln 3$

   h.) $4(1+10^{5x}) = 9$

   $10^{5x} = 1.25$

   $5x \log 10 = \log 1.25$

   $\ln 9 = 6$

4. Evaluate each expression.

a.) $\log 4 + \log 25$

$\log 4 + \log 25$

$\log 100 = 2$

b.) $\log_3 100 - \log_3 18 - \log_3 50$

$\log_3 18$

$\log_3 \frac{100}{18} - \log_3 50$

$\log_3 \frac{100}{900} = -2$

c.) $\log_4 16^{100}$

$\log_4 4^{2 \cdot 100}$

$= 200$

d.) $\log_3 5 + 5\log_3 2$

$\log_3 5 + \log_3 2^5$

$\log_3 (5 \cdot 2^5)$

$\log_3 160$

e.) $\ln 5 + 2\ln x - 2\ln(x^2 + 5)$

$\ln 5 + \ln x^2 - \ln(x^2+5)^2$

$\ln\left(\frac{5x^2}{(x^2+5)^2}\right)$

f.) $\log_2 8^{33}$

$\log_2 (2^3)^{33} = 99$

5. Use the laws of Logarithms to expand each expression.

a.) $\log_2(2x)$

$\log_2 2 + \log_2 x$

$1 + \log_2 x$

b.) $\log_5(x/2)$

$\log_5 x - \log_5 2$

c.) $\log_2(xy)^{10}$

$10(\log_2 x + \log_2 y)$

d.) $\sqrt[3]{\ln 3r^2 s}$

$(\ln 3r^2 s)^{1/3}$

$1/3(\ln 3 + 2\ln r + \ln s)$

6. The population of the world in 2000 was 6.1 billion and the estimated relative growth rate was 1.4% per year. If the population continues to grow at this rate, when will it reach 122 billion? Assume continuous growth.

$122 = 6.1 e^{.014t}$

$20 = e^{.014t}$

213 years

2213

7. A culture contains 1500 bacteria and doubles every 30 minutes.

a.) Find a function that models the number of bacteria after t minutes.

$A(t) = 1500(2)^{t/30}$

b.) Find the number of bacteria after 2 hours.

$A(4) = 1500(2)^4 = 24000$

c.) After how many minutes will the culture contain 4000 bacteria?

$4000 = 1500(2)^{t/30}$

8. The velocity of a skydiver t seconds after jumping is given by $v(t) = 80(1-e^{-0.2t})$. After how many seconds is the velocity 70 ft/sec?

$70 = 80(1 - e^{-.2t})$

$\frac{7}{8} - 1 = -e^{-.2t}$

$\frac{1}{8} = e^{-.2t}$

$\ln \frac{1}{8} = -.2t$

$t = 10.4$ sec.

9. A sum of $1000 was invested for 4 years, and the interest was compounded semiannually. If this sum amounted to $1453.77 in the given time, what was the interest rate?

$1000\left(1 + \frac{r}{2}\right)^{2 \cdot 4} = 1453.77$

$\left(1 + \frac{r}{2}\right)^{8} = (1.45377)^{1/8}$

$1 + \frac{r}{2} = 1.04789$

$\frac{r}{2} = .04789$

$r \approx 9.6\%$

10. Solve for x: $\log_2(\log_3 x) = 4$

$2^4 = \log_3 x$

$16 = \log_3 x$

$3^{16} = x$

43,046,721

11. The fox population in Blue Hills has a continuous growth rate of 8% per year. It s estimated that the population in 2000 was 5600.

a.) Find a function that models the population t years after 2000.

$A = 5600e^{.08t}$

b.) Use the function to estimate the number of foxes in 2010.

$A = 5600e^{.08(10)}$

$= 12463$ foxes

12. The population of California was 10,586,223 in 1950 and 23,668,562 in 1980. Assume the population grows exponentially.

a.) Find a function that models the population t years after 1950.

$23668562 = 10586223\, b^{30}$

$2.235 = b^{30}$

$1.0272 = b$

$A = 10586223(1.0272)^{t-1950}$

b.) Determine how long it takes the population to double.

$2 = 1.0272^{t-1950}$

$25.844 = t - 1950$

≈ 26 years

1975

c.) Use your function to determine the population of CA in 2000. How far is is from the actual population of 33,871,648?

40,503,408