

Chapter 2 - Polynomial Functions

- 1.) If $P(x) = -4x^2 + 5x - 1$, use synthetic substitution to find
 a.) $P(-2)$ b.) $P(1/3)$ c.) $P(i)$

$$\begin{array}{r|rrrr} -2 & -4 & 5 & -1 & \\ & 8 & -9 & 17 & \\ \hline & -4 & 13 & 16 & \end{array}$$

$$\begin{array}{r|rrrr} 1/3 & -4 & 5 & -1 & \\ & -4/3 & 11/3 & 1/3 & \\ \hline & -4 & 11/3 & 1/3 & \end{array}$$

$$\begin{array}{r|rrrr} i & -4 & 5 & -1 & \\ & -4i & 5i & -4i & \\ \hline & -4 & -4i & -1 & \\ & & -1 & 5i & \\ & & & 3 & \\ \hline & & & & 3+5i \end{array}$$

- 2.) One root of $3x^3 - 5x^2 - 3x + 2 = 0$ is 2. Find the remaining roots.

$$\begin{array}{r|rrrr} 2 & 3 & -5 & -3 & 2 \\ & 6 & 7 & 10 & \\ \hline & 3 & 1 & 7 & 12 \end{array}$$

$$3x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)} = \frac{-1 \pm \sqrt{13}}{6}$$

- 3.) Let $P(x) = 8x^3 - 6x - 1$.

$$\frac{194}{70}$$

- a.) Use synthetic division to find the quotient and remainder when $P(x)$ is divided by $x - 5$

$$\begin{array}{r|rrrr} 5 & 8 & 0 & -6 & -1 \\ & 40 & 200 & 970 & \\ \hline & 8 & 40 & 194 & -1 \end{array}$$

$$Q: 8x^2 + 40x + 194$$

$$R: -1$$

- b.) List all possible rational roots of the equation $P(x) = 0$

$$\begin{array}{l} \text{Factors 1} \\ \text{Factors 8} \end{array} \quad \begin{array}{l} \pm 1 \\ \pm 1, \pm 2, \pm 4, \pm 8 \end{array} \Rightarrow \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}$$

- c.) Is $x - 1$ a factor of $P(x)$?

$$\begin{array}{r|rrrr} 1 & 8 & 0 & -6 & -1 \\ & 8 & 8 & 2 & 1 \\ \hline & 8 & 8 & 2 & 0 \end{array}$$

No remainder $\text{int} = 0$

- 4.) Sketch the graph of $P(x) = -x(x+2)(x-1)^2$

degree = 4



- 5.) Let $P(x) = x^4 + 2x^3 + x^2 + 8x - 12$

- a.) Find the quotient and remainder when $P(x)$ is divided by $x + 2$

$$\begin{array}{r|rrrrrr} -2 & 1 & 2 & 1 & 8 & -12 \\ & -2 & -2 & 3 & -6 & 0 \\ \hline & 1 & 0 & 4 & 2 & 0 \end{array}$$

$$Q: x^3 + x^2 + 4x + 2$$

$$R: 0$$

- b.) Show that $x - 1$ is a factor of $P(x)$.

$$\begin{array}{r|rrrrrr} 1 & 1 & 2 & 1 & 8 & -12 \\ & 1 & 3 & 4 & 12 & 0 \\ \hline & 1 & 3 & 4 & 12 & 0 \end{array}$$

remainder = 0

- c.) Find all the rational zeros.

$$\begin{array}{r|rrrr} -3 & 1 & 2 & 1 & 8 & -12 \\ & -3 & -4 & -5 & -7 & 0 \\ \hline & 1 & -1 & -4 & 1 & 0 \end{array}$$

$$y^2 + 4 = 0$$

$$y^2 = -4$$

$$x = \pm 2i, -3, 1$$

- d.) Find $P(i)$

$$\begin{array}{r|rrrrrr} i & 1 & 2 & 1 & 8 & -12 \\ & i & 2i & -1 & 8i & -12i \\ \hline & 1 & 3i & 0 & 8i & -24i \end{array}$$

- e.) Solve $P(x) = 0$ giving all imaginary and real roots.

- 6.) Show that $x^3 + 2x^2 + 4 = 0$ has no rational root but does have an irrational root. Then approximate

$$x \approx -1.1 \quad \text{or} \quad x \approx 2.7 + 1.8i \quad (1.43)$$

Chapter 4 Functions

- 1.) Give the domain of each function

a.) $f(x) = \frac{x-1}{x^2-4x+3}$

b.) $f(x) = \sqrt{x^2-4}$

$P(x) = \frac{x-1}{(x-3)(x-1)}$ D: $\{x \mid x \neq 3, 1\}$

D: $\{x \mid x \geq 2 \text{ or } x \leq -2\}$

- 2.) Let $f(x) = x^2 + 6x + 5$. Find each of the following.

a.) $(f+h)(x)$

b.) $g^{-1}(x)$

c.) $(f \circ g)(x)$

$P(x) + h(x)$

$x^2 + 6x + 5 + x^2 - 3$

$2x^2 + 6x + 2$

$g^{-1}(x)$

$x = 7y - 2$

$\frac{y+2}{7} = g^{-1}(y)$

$g(g(x))$

$g(7x-2)$

$(7x-2)^2 + 6(7x-2) + 5$

$49x^2 + 28x + 4 + 42x - 12 + 5$

$49x^2 + 70x - 3$

- 3.) Let $f(x) = 2x^2 - x^4$.

- a.) Find the domain, range and zeros of the function.

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \leq 1\}$

Zeros: $x = \pm \sqrt{2}, 0$ (or)

- b.) Describe the graph of $x = f(y)$. Does $x = f(y)$ represent a function?

Reflects over line $y = 0$

No fails the vertical line test (new graph) original fails horizontal line test
(x values are not unique)

- 4.) When a ball is thrown upward, its approximate height in meters t seconds later is given by

$h(t) = 36 + 24t - 5t^2$. Find the domain, range and zeros of h .

D: $\{t \mid 0 \leq t \leq 6\}$

R: $\{h(t) \mid 0 \leq h(t) \leq 64\}$

$0 = -5t^2 + 24t + 36$
 $t = \frac{-24 \pm \sqrt{24^2 - 4(-5)(36)}}{2(-5)} = \frac{-24 \pm \sqrt{1296}}{-10} = \frac{-24 \pm 36}{-10} = 6 \text{ or } -1.2$

- 5.) Let $f(x) = .5x - 5$. Find $f^{-1}(x)$ and show that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x)$.

$y = \frac{1}{2}x - 5$

$2x + 10 = f^{-1}(y)$

$x = \frac{1}{2}y - 5$

$f(2x + 10) = \frac{1}{2}(2x + 10) - 5 = x$

$x + 5 = \frac{1}{2}y$

$f^{-1}(\frac{1}{2}x - 5) = 2(\frac{1}{2}x - 5) + 10 = x$

- 6.) Describe the graph of $y^2 - 4x^2 = 4$. Then tell if the graph has symmetry in the (a) x-axis, (b) y-axis, (c) the line $y=x$, and/or (d) the origin.

hyperbola

$\frac{y^2}{4} - \frac{x^2}{1} = 1$



$y^2 = 4x^2 + 4$

x-axis: $(x, y) \rightarrow (x, -y)$

$(-y)^2 - 4x^2 = 4$ ✓

y-axis: $(x, y) \rightarrow (-x, y)$

$y^2 - 4(-x)^2 = 4$ ✓

$y = x$: $(x, y) \rightarrow (y, x)$

$x^2 - 4y^2 = 4$ ✗

origin: $(x, y) \rightarrow (-x, -y)$

Chapter 7/8 Trigonometric Functions and Equations

- 1.) Name two angles, one positive and one negative, that are coterminal with the given angle.
 a.) 110° $110 - 360 = -250^\circ$ b.) 2.3 $2.3 - 2\pi = -3.78$
 470° $2.3 + 2\pi = 8.58$

- 2.) Given $\sin x = 0.6$ and $\cos x < 0$, find the value of the following without using a calculator.
 a.) $\cos x = \frac{-8}{10} = -\frac{4}{5}$ b.) $\tan x = \frac{-6}{8} = -\frac{3}{4}$

- 3.) Find the exact value of the expression.
 a.) $\sin 270^\circ = -1$ b.) $\cos 300^\circ = \frac{1}{2}$

- c.) $\sin^{-1}(0.5) = 60^\circ \text{ or } 120^\circ$ d.) $\tan^{-1}(-1) = 135^\circ \text{ } 315^\circ$

- 4.) If angle A is obtuse and $\cos A = (-1/3)$, find $\sin A$ and $\tan A$.

$$\sin A = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3} \quad \tan A = -\frac{\sqrt{8}}{1} = -2\sqrt{2}$$

- 5.) Solve $0^\circ \leq x < 360^\circ$. Give answers to the nearest tenth of a degree.

- a.) $4\tan x - 1 = 0$ b.) $\sin x \cos x = \sin x$

$$\tan x = 1/4$$

$$x = 14^\circ \text{ or } 194^\circ$$

$$\sin x (\cos x - 1) = 0$$

$$\sin x = 0 \quad \cos x = 1$$

$$180^\circ \quad 0, 360^\circ$$

- c.) $(2\sin x - 1)(3\cos x - 1) = 0$ d.) $2\sin 2x = 3(1 - \cos x)$

$$\sin x = 1/2 \quad \cos x = 1/3$$

$$30^\circ \text{ } 150^\circ \quad 70.5^\circ \text{ } 289.5^\circ$$

$$4.4 \quad 0.180$$

- 6.) Give the amplitude and period of each function.

- a.) $y = 1.5\sin 2x$

$$A = 1.5$$

$$\frac{2\pi}{P} = 2 \quad P = \pi$$

- b.) $y = 2 - 3\cos \pi x$

$$A = 3 \text{ (always +)}$$

$$\frac{2\pi}{P} = \pi \quad P = 2$$

- 7.) Simplify

- a.) $\frac{\cot x + \tan x}{\tan x}$

$$\frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}}{\frac{\sin x}{\cos x}} = \frac{1}{\sin^2 x} = \csc^2 x$$

- b.) $(\cos 2x)/(1 - \sin x)$

$$\frac{1 - 2\sin^2 x}{1 - \sin x}$$

- 8.) Find the area of $\triangle ABC$ if $a = 10$, $b = 17$, $\angle C = 100^\circ$.

$$\text{area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} (10)(17) \sin 100$$

$$= 83.7 \text{ units}$$

- 9.) Two ships leave port at 2 PM. One travels due east at 20 knots and the other on a course of 200° at 30 knots.

- a.) How far apart are the ships at 4 PM?

$$c^2 = 60^2 + 40^2 - 2(60)(40) \cos 110$$

$$c^2 = 6841.7$$

- b.) Where will they meet at midnight??

$$= 82.7 \text{ nautical miles}$$