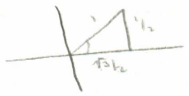
Practice Problems
Polar and Complex Numbers

Name _____

1. If $z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$, express z in polar form.

Calculate z^2 and z^3 by using De Moivre's theorem.



1 cis 30

z^2 = 1 cis 60 z^3 = 1 cis 90

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2. Compute $(1 - i\sqrt{3})^5$. Express your answer in polar and rectangular form.

2 cis 300

$z^5 = 2^5$ cis 300 · 5

= 32 cis 1500 = 32 cis 60 = 32(cis 60 + i sin 60) = 16 + 16$\sqrt{3}i$

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3. Find all three cube roots of 8i. Call them z_1, z_2, and z_3.
Express your answers in rectangular form.

$8i$ = 8 cis 90 $-i$

z_1 = 2 cis 30 $\sqrt{3} + i$

z_2 = 2 cis 150 $-\sqrt{3} + i$

z_3 = 2 cis 270 $-2i$

$z^3 = r^3$ cis θ

8 cis 90 = r^3 cis 30

$r^3 = 8$ $r = 2$ $\theta = \frac{90}{3} = 30$

$2\text{cis}\left(\frac{90}{3} + 0\right)$

$2\text{cis}\left(\frac{90}{3} + \frac{360}{3}\right) = 2\text{cis}150$

$2\text{cis}\left(\frac{90}{3} + \frac{2\cdot360}{3}\right) = 2\text{cis}270$

Find the sum: $z_1 + z_2 + z_3$ = 0

Find the product: $z_1 \cdot z_2 \cdot z_3$ = 8i

$(\sqrt{3}+i)(-\sqrt{3}+i)$

$(-3-1)-2i = -4\cdot-2i = 8i$

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4. Find all three cube roots of -8. Call them z_1, z_2, and z_3.
Express your answers in rectangular form.

$-8 = 8$ cis 180

z_1 = 2 cis 60 $1 + \sqrt{3}i$

z_2 = 2 cis 180 -2

z_3 = 2 cis 300 $1 - \sqrt{3}i$

$z^3 = r^3$ cis 3θ

8 cis 180 = r^3 cis 3θ

$r = 2$ $3\theta = 180$ $\theta = 60$

$2\text{cis}\left(\frac{180}{3} + \frac{360}{3}\right)$

$2\text{cis}\,180$

$2\text{cis}\,\frac{180}{3} + \frac{720}{3}$

$2\text{cis}\,300$

Find the sum: $z_1 + z_2 + z_3$ = 0

Find the product: $z_1 \cdot z_2 \cdot z_3$ = -8

$(1+\sqrt{3}i)(1-\sqrt{3}i)(-2)$

$(1-2i-3)(-2)$