

Worksheet - Vectors in 3D

- 1) Given
- $A = (2, 8, -5)$
- and
- $B = (2, 0, -3)$
- determine:

(2)

a) $|\overrightarrow{AB}|$ $\overrightarrow{AB} = (0, -8, -2)$

$$\sqrt{0^2 + 8^2 + 2^2} = \sqrt{68}$$

- b. midpoint of
- \overline{AB}

$$\left(\frac{2+2}{2}, \frac{8+0}{2}, \frac{-5+(-3)}{2} \right) = (2, 4, -4)$$

- 2) Given
- $A = (4, 1, 2)$
- and
- $B = (-4, -1, -2)$
- determine:

(4)

a) $|\overrightarrow{AB}|$

$$\overrightarrow{AB} = (-8, -2, -4)$$

$$\sqrt{64 + 4 + 16} = \sqrt{84} = 2\sqrt{21}$$

- b. midpoint of
- \overline{AB}

$$\left(\frac{4+(-4)}{2}, \frac{1+(-1)}{2}, \frac{2+(-2)}{2} \right) = (0, 0, 0)$$

- 3) Simplify each expression.

(8)

a) $(8, 7, 4) - (2, 0, 9) + (1, -2, 1)$

$$(6, 7, -5) + (1, -2, 1)$$

$$(7, 5, -4)$$

b. $(1, 4, 2) \cdot (3, 1, 2)$

$$3 + 4 + 4 = 11$$

c. $|(6, 2, 3)|$

$$\sqrt{36 + 4 + 9} = \sqrt{49} = 7$$

- 4) Are vector
- $\mathbf{u} = (3, -7, 1)$
- and vector
- $\mathbf{v} = (6, 3, 3)$
- perpendicular?

$$\mathbf{u} \cdot \mathbf{v} = 0?$$

$$18 - 21 + 3 = 0 \quad \text{yes}$$

- 5) Determine the value of
- k
- if
- $(2, k, -3)$
- and
- $(4, 2, 6)$
- are perpendicular.

$$8 + 2k - 18 = 0$$

$$2k = 10$$

$$k = 5$$

- 15.) 6) Given
- $\mathbf{u} = (8, 6, 0)$
- and
- $\mathbf{v} = (2, -1, 2)$
- , determine the angle between the vectors.

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|} = \frac{16 - 6}{\sqrt{64 + 36} \cdot \sqrt{4 + 1 + 4}} = \frac{10}{10 \cdot \sqrt{9}} = \frac{1}{3}$$

- 7) Given
- $\mathbf{u} = (2, 2, 1)$
- and
- $\mathbf{v} = (3, 6, -2)$
- , determine the angle between the vectors.

$$\frac{6 + 12 - 2}{\sqrt{9} \cdot \sqrt{49}} = \frac{16}{3 \cdot 7} = \frac{16}{21} = \cos \theta$$
$$\theta = 48.71^\circ$$

8) Line L has vector equation $(x, y, z) = (-2, 0, 1) + t(4, -1, 1)$.

a) List the three parametric equations of L .

$$\begin{aligned} x &= -2 + 4t & z &= 1 + t \\ y &= -t \end{aligned}$$

b) Name two points on L .

$$(-2, 0, 1)$$

$$(2, -1, 2)$$

c) Which of the points $(-10, 2, -1)$, $(18, -2, 6)$, and $(14, -5, 4)$ are on L ?

$$\begin{array}{ccc} t = -2 & t = 5 & t = 4 \\ (-10, 2, -1) & (18, -2, 6) & (14, -5, 4) \end{array}$$

$$-10 = -2 + 4t$$

$$-8 = 4t$$

$$-2 = t$$

$$y = 2$$

$$z = 1$$

$$18 = -2 + 4t$$

$$20 = 4t$$

$$5 = t$$

$$y = -5$$

d) Write a vector equation of the line containing $(1, 2, 3)$ that is parallel to L .

$$(x, y, z) = (1, 2, 3) + t(4, -1, 1)$$

9) Write vector and parametric equations for the line containing $A(2, 3, 1)$ and $B(4, -1, 3)$

$$(x, y, z) = (2, 3, 1) + t(2, -4, 2)$$

10) Where does the line $(x, y, z) = (6, -5, 4) + t(3, -2, 4)$ intersect

a) The xy -plane

$$z = 0$$

$$0 = 4 + 4t \quad t = -1 \quad (3, -3, 0)$$

$$x = 6 + 3t$$

$$y = -5 - 2t$$

$$z = 4 + 4t$$

b) The yz -plane

$$x = 0$$

$$0 = 6 + 3t$$

$$-2 = t$$

$$(0, -1, -4)$$

c) The xz -plane

$$y = 0$$

$$-5 - 2t = 0$$

$$t = -5/2$$

$$(-3\frac{1}{2}, 0, -6)$$