

Name: W

614 Applications of Determinants

1. Evaluate each determinant. Do NOT use a calculator. Show all work.

a.) $\begin{vmatrix} 3 & 7 \\ -1 & 4 \end{vmatrix}$

$12 - (-7) = 19$

b.) $\begin{vmatrix} -7 & 1 & 3 \\ 2 & -1 & 5 \\ 3 & 0 & 2 \end{vmatrix}$

$14 + 15 + 19 - 4 = 34$

2. Solve using Cramer's Rule

a.) $\begin{cases} 5x + 7y = 8 \\ -2x - 9y = -2 \end{cases}$

$D = \begin{vmatrix} 5 & 7 \\ -2 & -9 \end{vmatrix} = -45 - (-14) = -31$

$D_x = \begin{vmatrix} 8 & 7 \\ -2 & -9 \end{vmatrix} = -72 + 14 = -58$

$x = \frac{-58}{-31} = \frac{58}{31}$

b.) $\begin{cases} 3x + 2y - z = -6 \\ x - 3y + 2z = 2 \\ 5x - 6z = -7 \end{cases}$

$\left(\frac{-91}{-11}, \frac{-73}{-11}, \frac{7}{-11} \right)$

3. Find all the values of x for which

$\begin{vmatrix} x-4 & 0 & 0 \\ 0 & x+4 & 0 \\ 0 & 0 & x+1 \end{vmatrix} = 0$

$(x-4)(x+4)(x+1) = 0$
 $(x^2 - 16)(x+1)$

$x = 4, -4, -1$

4. Use the fact that the equation of a line passing through (x_1, y_1) and (x_2, y_2) can be written as

$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} x & y & 1 \\ -3 & -1 & 1 \\ 2 & 9 & 1 \end{vmatrix} = 0$

$(-1-9)x + y(-3-2) - 27+2$

$-10x - 5y = 25$

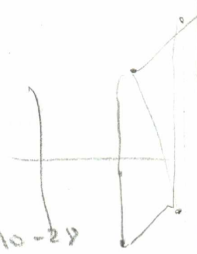
$y = 2x + 5$

to find the equation of the line passing through $(-3, -1)$ and $(2, 9)$. Write your equation in slope intercept form.

5. Use determinants to find the area of the parallelogram whose corners are located at A (2,6), B(7,10) and C(9, -2).

$\vec{AB} = (5, 4)$

$\begin{vmatrix} 5 & 4 \\ 7 & -8 \end{vmatrix} = -40 - 28 = -68$



6. Three vectors $\mathbf{x} = (4, 5, 2)$, $\mathbf{w} = (6, -1, 4)$ and $\mathbf{q} = (-3, 5, 2)$ determine a parallelepiped. Using determinants, find its volume.

$$\det \begin{vmatrix} 4 & 5 & 2 \\ 6 & -1 & 4 \\ -3 & 5 & 2 \end{vmatrix} = 15403$$

7. Using determinants, find a vector together with $\mathbf{v} = (4, -8)$ that will determine a parallelogram with an area of 100 square units.

$$\begin{vmatrix} 4 & -8 \\ x & y \end{vmatrix} = 100$$

$$x = 8$$

$$4y + 8x = 100$$

one possible answer

$$4y + 64 = 100$$

$$\langle 8, 9 \rangle$$

8. Find the equation of a plane containing the point $(3, -5, 9)$ that is perpendicular to the vector $\mathbf{m} = (-2, 4, 7)$.

$$-2x + 4y + 7z = d$$

$$-6 - 20 + 63 = d$$

$$37 = d$$

$$-2x + 4y + 7z = 37$$

9. Find the area of a triangle formed by the points A $(3, -5)$ B $(2, 6)$ and C $(-3, 5)$

$$\overrightarrow{AB} = (-1, 11)$$

$$\overrightarrow{AC} = (-6, 10)$$

$$\frac{1}{2} \begin{vmatrix} -1 & 11 \\ -6 & 10 \end{vmatrix}$$

$$= \frac{1}{2} (56) = 28$$

10. Solve each system using Cramer's rule.

a.) $\begin{cases} x + 2y = 3 \\ 5x + 10y = 6 \end{cases}$

same line

b.) $\begin{cases} 5x - y = -13 \\ 5x - y = -12 \end{cases}$

infinite solutions

$$N_1 \begin{vmatrix} 5 & -13 \\ 5 & -12 \end{vmatrix} = -60 + 65 = 5$$

- c.) Explain how to interpret the results of the values of the determinants