Part I

1.) A line passes through point A(–1, 3) and has a directional vector in component form of (2, 5). Determine the equation of the vector. Write the equation for the vector in parametric form.

$(x, y) = (-1, 3) + t(2, 5)$

$x = -1 + 2t$

$y = 3 + 5t$

2.) Write the vector equation of the line that passes through the points (1,2) and (–2, 5). Write the equation of the vector in parametric form.

$\overrightarrow{AB} = (-2-1, 5-2) = (-3, 3)$

$(x, y) = (1, 2) + t(-3, 3)$

$x = 1 + -3t$

$y = 2 + 3t$

3.) A particle moves along a line in the coordinate plane.

a.) Determine the velocity and speed of the moving point.

b.) Determine the parametric equations of the moving point.

i.) (x,y)=(1,4)+t(3, –2)

Velocity $(3, -2)$

Speed $|\vec{v}| = \sqrt{9+4} = \sqrt{13}$

$x = 1 + 3t$

$y = 4 - 2t$

ii.) (x,y) = (–2,0) + t(1, 3)

Velocity $(1, 3)$

Speed $= \sqrt{10}$

$x = -2 + t$ $y = 0 + 3t$

3. Determine the vector and parametric equations of the moving object with velocity (1, –1) and position at t=0 is (1, –5)

$(x, y) = (1, -5) + t(1, -1)$

$x = 1 + t$

$y = -5 - t$

4. A line has a vector equation (x,y) = (3, 2) + t (2, 4). Give a pair of parametric equations and a Cartesian equation for the line.

$x = 3 + 2t$

$y = 2 + 4t$

$\frac{x-3}{2} = t$

$y = 2 + 4\left(\frac{x-3}{2}\right)$

$y = 2 + 2x - 6$ $y = 2x - 4$

5. At time t, the position of an object moving with constant velocity is given by the parametric equations x = 2-3t and y = –1 + 2t.

a.) Determine the velocity and the speed of the object.

$(x, y) = (2, -1) + t(-3, 2)$

$\vec{v} = (-3, 2)$

Speed $= \sqrt{13}$

b.) Where and when does it cross the line x + y = 2?

$2 - 3t + -1 + 2t = 2$

$x = 2 - 3(-1) = 2 + 3 = 5$

6. An object moves with a constant velocity so that its position at time t is $(x,y) = (1,1) + t(-1, 1)$. When and where does the object cross the circle $(x-1)^2 + y^2 = 5$

$x = 1 + -t \qquad y = 1 + t$

$t = -2 \quad (3, -1)$

$t = 1 \quad (0, 2)$

$(1 + -t - 1)^2 + (1 + t)^2 = 5$

$t^2 + 1 + 2t + t^2 = 5$

$2t^2 + 2t + -4 = 0$

$2(t^2 + t - 2) = 0$

$(t+2)(t-1) = 0$

$t = -2$ or 1

7. Given the points A (–1,1) and B(2,7). Find the coordinates of the point P that is 1/6 of the way from A to B.

$\overrightarrow{AB} = (3, 6)$

$\frac{1}{6}(3, 6) = (\frac{1}{2}, 1)$

$P = (-1, 1) + (\frac{1}{2}, 1) = (-\frac{1}{2}, 2)$

Part II

0.) Determine if the following pairs of vector equations collide (hit at the same point at the same time.) $(2, 7)$

a.) $(x,y) = (3,5) + t(-2,4)$ and $(x,y) = (5,4) + t(-6, 6)$

$x = 3 - 2t \qquad x = 5 - 6t$

$y = 5 + 4t \qquad y = 4 + 6t$

$x_1 = x_2$

$3 - 2t = 5 - 6t$

$4t = 2$

$t = 1/2$

$y_1 = 7 \qquad y_2 = 7$ yes

b.) $(x,y) = (1,2) + t(-3, 2)$ and $(x,y) = (2,4) + t(1,1)$

$x_1 = 1 + -3t \qquad x_2 = 2 + t$

$y_1 = 2 + 2t \qquad y_2 = 4 + t$

$1 + -3t = 2 + t$

$-1 = 4t$

$-1/4 = t$

$y_1 = 1\frac{1}{2} \qquad y_2 = 3\frac{3}{4}$ no

1.) Tanya, who is a long distance runner, runs at an average velocity of 8 miles per hour. Two hours after Tanya leaves, you leave your house in your Honda and follow the same route. If your average velocity is 40 miles per hour, how long will it take you to catch up to Tanya?

Use a simulation of two motions and parametric equations to verify the answer.

$t \to$ time Tanya leaves

Tanya $x = 8t$ Honda $x = 40(t - 2)$

catch up when $x_1 = x_2$

$8t = 40(t - 2)$

$80 = 32t$

$\frac{80}{32} = \frac{40}{16} = 2.5 = t$

2.5 hours

2.) Two cars start at the origin at time t=0.

Car A travels North at a speed of 50mph

Car B travels West at a speed of 60 mph

a.) Give each car's parametric equation for x and y positions (miles) s a function of time (hours.)

A $\to (x, y) = (0, 0) + t(0, 50)$ $\qquad x = 0 \qquad y = 50t$

$t(-60, 0)$ $\qquad x = -60t \qquad y = 0$

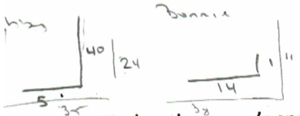
b.) How far were the two cars away from each other after 2 hours?

$t = 2$ A= 100 B = −120

$d = \sqrt{100^2 + 120^2} = 156.2$ miles

3.) Abby and Bonnie are riding their bikes, each at a constant speed in straight lines.
At t=0 seconds Abby is 5 feet East and 40 feet North of home, and Bonnie is 14 feet East and 1 foot North of home.
At t=2 seconds, Abby is 35 feet East and 24 feet North of home, and Bonnie is 38 feet East and 11 feet North of home.

a.) Find the horizontal and vertical components of each person's velocity in feet per second.



Abby $\frac{1}{2}(30, -16) = (15, -8)$ H 15 ft/s V −8 ft/sec

Bonnie $\frac{1}{2}(24, 10) = (12, 5)$ H 12 ft/s V 5 ft/s

b.) Find each person's speed in feet per second.

Speed $|\vec{v}| = \sqrt{225 + 64} = 17$ ft/s Abby

$= \sqrt{144 + 25} = 13$ ft/s Bonnie

c.) Give the parametric equations for the x and y position of each person's bike (in feet) as a function of time (in seconds.)

$X_A = 5 + 15t$ $Y_A = 40 - 8t$

A $(x, y) = (5, 40) + t(15, -8)$

B $(x, y) = (14, 1) + t(12, 5)$ $X_B = 14 + 12t$ $Y_B = 1 + 5t$

d.) Do the two girls collide? In order to support your answer you must show your work.

$X_A = X_B$ $5 + 15t = 14 + 12t$ $3t = 9$ $t = 3$

Y_A @ $t = 3$ $40 - 24 = 16$

Y_B @ $t = 3$ $1 + 15 = 16$ yes at $t = 3$

4.) Dominique is traveling at a constant speed in a straight line. At t=5 seconds, she is at the point (200,100). At t=9 seconds, she is at the point (160, 120). Give the parametric equation for the x and y position of Dominique (in feet) as a function of time (in seconds.) Use (200, 100) as a starting position and manipulate the parameter to account for the time. (50, 16)

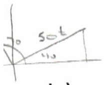
$\vec{v}$ $(-40, 20)$ $\vec{v}$ $(-10, 5)$

$(x, y) = (200, 100) + (t-5)(-10, 5)$

$x = 200 - 10(t-5)$ $y = 100 + 5(t-5)$

5.) Two cars start at the origin at t = 0.
Car A travels 50 mph in the standard position direction of 40°.
Car B travels 60 mph in the standard position of 110°

a.) Give each car's parametric equation for x and y positions (in miles) as a function of time (hours.)



$X_A = (50\cos 40)t$ $X_B = (60\cos 110)t$

$Y_A = (50\sin 40)t$ $Y_B = (60\sin 110)t$

b.) How far were the two cars away from each other after 1 hour?

$= 50^2 + 60^2 - 2 \cdot 50 \cdot 60 \cdot \cos(110-40)$

≈ 76

c.) **After how many seconds were they 100 miles apart?**

$$100^2 = \sqrt{(60t)^2 + (50t)^2 - 2(60t)(50t)\cos 70}$$

LOL

$$100^2 = 2500t^2 + 3600t^2 - 15000t^2 \cos 70$$

$$4048t^2 = 10000$$

$$t^2 = 2.47 \qquad t \approx 1.57 \text{ hours}$$

Ga

vector

$$100 = \sqrt{((60\cos 110)t - (50\cos 40)t)^2 + ((60\sin 110)t - (50\sin 40)t)^2}$$

$$100 = \sqrt{(-20.52t - 38.3t)^2 + (56.38t - 32.14t)^2}$$

$$100 = \sqrt{(58.82t)^2 + (24.24)^2}$$

$$100 = \sqrt{3459.8t^2 + 587.6t^2}$$

$$100 = \sqrt{4047t^2}$$

$$10000 = 4047t^2$$

$$2.47 = t^2 \qquad t = 1.57$$

north

y) su

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1^2 -

2(8)

75