614 Review Chapter 12 part II Name: ____________________

1. At time t, the position of an object moving with constant velocity is given by the parametric equations x = 1 + 5t and Y = –1 + 12t

a.) Find the velocity and the speed of the object

$(x,y) = (1,-1) + t(5,12)$

Velocity $(5,12)$

Speed 13

b.) Where and when does the object cross the line 2x – y = 5?

$2(1+5t) - (-1+12t) = 5$

$2 + 10t + 1 - 12t = 5$

$-2t + 3 = 5$

$-2t = 2$

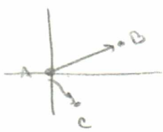
$t = -1$

$x = 1 - 5 \Rightarrow -4$

$y = -1 - 12 \Rightarrow -13$

$(-4, 13)$

2. Given A (0,0), B (4, 3) and C (1, –2) find the approximate measure of angle BAC.



$\overrightarrow{AB}$ $(4,3)$

$\overrightarrow{AC}$ $(1,-2)$

$\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}| \cdot |\overrightarrow{AC}|}$

$\cos\theta = \dfrac{4\cdot 1 + 3\cdot -2}{\sqrt{4^2+3^2} \cdot \sqrt{1+(-2)^2}} \Rightarrow \dfrac{-2}{5\sqrt{5}} = \cos\theta$

100.3

3. Find a vector equation of a line through (–3, –1) and parallel to the line (x,y)= (2, 7) + t (1, 5).

$(x,y) = (-3,-1) + t(1,5)$

Same direction vector

4. If u = (5, 7, –3) and v = (–2, –4, 0) , find $\vec{u} \cdot \vec{V}$.

$\vec{u} \cdot \vec{v} = 5 \cdot -2 + 7 \cdot -4 + -3 \cdot 0$

$= -10 - 28$

$= -38$

5. Find $\|\overline{AB}\|$, if A (4, –3, –1) and B (0, 1, –1).

$\overrightarrow{AB}$ $(-4, 4, 0)$

$|\overrightarrow{AB}| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

6 if A = (8, –5, 7) and B = (3, 4, 9)

a.) .Find the midpoint of segment AB

$$\left(\frac{8+3}{2}, \frac{-5+4}{2}, \frac{7+9}{2}\right)$$

$$\left(\frac{11}{2}, -\frac{1}{2}, 8\right)$$

b.) Determine $\overrightarrow{AB}$ then find $\|\overrightarrow{AB}\|$.

$\overrightarrow{AB} = (-5, -9, 2)$

$|\overrightarrow{AB}| = \sqrt{25+81+4} \quad = \sqrt{110}$

7. Find a vector and parametric equations for the line containing A (5, –2, 4) and B (6, 1, 1).

$(x, y, z) = (5, -2, 4) + t(1, 3, -3)$

or $(x, y, z) = (6, 1, 1) + t(-1, -3, 3)$

8. Find the measure of the angle between the vectors (1, 0, –1) and (3, –5, –4).

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} \Rightarrow \frac{1\cdot 3 + 0\cdot -5 + -1\cdot -4}{\sqrt{1^2+1^2}\cdot\sqrt{9+25+4}} = \frac{3+4}{\sqrt{2}\,\sqrt{38}} = \frac{7}{\sqrt{76}}$$

$$\cos\theta = \frac{7}{\sqrt{76}}$$

9. Line L has a vector equation (x, y, z) = (3, –4, –2) +t(–1, 1, 5)

a.) Name two points on L.

$(3, -4, -2)$ $\quad (1, -2, 8)$

$(2, -3, 3)$

b.) Write a vector equation through (–1, 0 , 7) and parallel to L.

$(x, y, z) = (-1, 0, 7) + t(-1, 1, 5)$

c.) Is the line with vector equation (x, y, z) = (3, –4, –2) + t(2, 5, –2) perpendicular to the original line? Explain.

$\vec{u}\cdot\vec{v} = 0$?

$(-1, 1, 5)\cdot(2, 5, -2)$

$-1\cdot 2 + 1\cdot 5 + 5\cdot -2$

10. Solve the system using Cramer's rule. You must show your work.
$5x + 3y = 8$
$2x + y = 4$

$D = \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} = 5 - 6 = -1$

$N_x = \begin{vmatrix} 8 & 3 \\ 4 & 1 \end{vmatrix} = 8 - 12 = -4$

$N_y = \begin{vmatrix} 5 & 8 \\ 2 & 4 \end{vmatrix} = 20 - 16 = 4$

$x = \frac{N_x}{D} = \frac{-4}{-1} = 4$

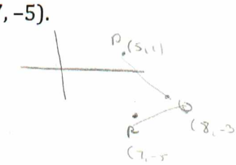
$y = \frac{N_y}{D} = \frac{4}{-1} = -4$

$(4, -4)$

11. Find the area of a triangle with vertices P (5, 1) Q(8, –3) and R (7, –5).

$\overrightarrow{PQ}$ $(3, -4)$
$\overrightarrow{QR}$ $(-1, -2)$

$\frac{1}{2}\text{abs}\begin{vmatrix} 3 & -4 \\ -1 & -2 \end{vmatrix}$



$= \frac{1}{2}\text{abs}(-6 - 4) = \frac{1}{2} \cdot 10\ \text{un}^2 = 5\ \text{un}^2$

12. Solve the system using Cramer's Rule
$x + 3y - z = 8$
$2x + y + 4z = -12$
$-x - 5y + 2z = 0$

$\left(\frac{188}{7}, \frac{-76}{7}, \frac{-96}{7}\right)$

$D = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 1 & 4 \\ -1 & -5 & 2 \end{vmatrix} = 7$

$N_x = \begin{vmatrix} 8 & 3 & -1 \\ -12 & 1 & 4 \\ 0 & -5 & 2 \end{vmatrix}\begin{matrix} 8 & 3 \\ -12 & 1 \\ 0 & -5 \end{matrix}$

$16 + 0 + -60 - 0 - -160 + 72 = 188$

$N_y = \begin{vmatrix} 1 & 8 & -1 \\ 2 & -12 & 4 \\ -1 & 0 & 2 \end{vmatrix}\begin{matrix} 1 & 8 \\ 2 & -12 \\ -1 & 0 \end{matrix}$

$-24 + -32 + 12 - 0 - 32 = -76$

$N_z = \begin{vmatrix} 1 & 3 & 8 \\ 2 & 1 & -12 \\ -1 & -5 & 0 \end{vmatrix}\begin{matrix} 1 & 3 \\ 2 & 1 \\ -1 & -5 \end{matrix}$

$0 + 36 - 80 + (-8) - 60 - 0 = -96$

$x = \frac{N_x}{D} = \frac{188}{7}$

$y = \frac{N_y}{D} = \frac{-76}{7}$

$z = \frac{N_z}{D} = \frac{-96}{7}$

13. Find the value of a if the vectors (6, –10) and (4, a) are
a.) parallel
b.) perpendicular

a.) $\frac{-10}{6} = \frac{a}{4}$

$-40 = 6a$

$\frac{-40}{6} = a$

$-20/3 = a$

b.) $6 \cdot 4 + -10 \cdot a = 0$

$24 + -10a = 0$

$24 = 10a$

$a = 2.4$

14. The three vectors **u** = (4, 5,7); **w** = (2, –1, 4) and **q** = (–3, 5, 2) determine a 3D parallelogram called a parallelpiped. Find the volume.

$\text{abs}\begin{vmatrix} 4 & 5 & 7 \\ 2 & -1 & 4 \\ -3 & 5 & 2 \end{vmatrix}\begin{matrix} 4 & 5 \\ 2 & -1 \\ -3 & 5 \end{matrix} = -8 + -60 + 70 - 21 - 80 - 20$

$= \text{abs}\,|-119|$