Name: _____ **623 Review Exponents and Logarithms**

NEATLY – SHOW ALL WORK!

1. Graph $y = 2^x$ and it's inverse.

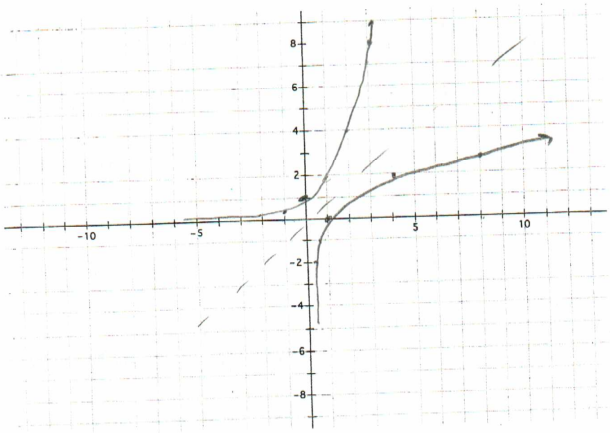
   a.) Which point must each graph contain? (They are different.)

   $y = 2^x$ $(0,1)$ $\log_2 y = x$ $(1,0)$

   b.) What is the domain of each function? The range?

   exp D: $\mathbb{R}$ R $y > 0$ log D $x > 0$ R $\mathbb{R}$

   c.) Does either function have any asymptotes? If so, give the equations.



2. Solve each equation for x.

(a) $\log_3 x = 9$ $3^9 = x$

(b) $\log_2 8 = x$ $2^x = 8$ $x = 3$

(c) $\log_3 27 = x$ $3^x = 27$ $x = 3$

3. Find x in each of the following:

(a) $\ln x = 2.7$ $e^{2.7} = x$ $x = 14.9$

(b) $\ln(x+1) = 1.86$ $e^{1.86} = x+1$ $x = 5.42$

(c) $x = e^{9.8} \div e^{7.6}$ $x = \frac{e^{9.8}}{e^{7.6}} = e^{(9.8-7.6)} = e^{2.2} = 9.03$

(d) $6.27 = e^x$ $x = \ln 6.27$ $x = 1.83$

(e) $4.12 = e^{-2x}$ $-2x = \ln 4.12$ $x = \frac{\ln 4.12}{-2} = -.7079$

4. Solve each equation for the variable.

(a) $y = 3^4$ $y = 81$

(b) $27 = 3^x$ $x = 3$

(c) $m = 4^2$ $m = 16$

5.

Solve the following:

(a) $\log_3 x = 4$ $3^4 = x$ $x = 81$

(b) $\log_m 81 = 4$ $m^4 = 81$ $m = 3$

(c) $\log_x 1000 = 3$ $x = 10$

(d) $\log_2 \frac{x}{2} = 5$ $2^5 = \frac{x}{2}$ $32 \cdot 2 = x$ $x = 64$

(e) $\log_3 y = 5$ $y = 3^5$ $y = 243$

(f) $\log_2 4x = 5$ $2^5 = 4x$ $\frac{32}{4} = x$ $8 = x$

6.

Simplify the following

(a) $\log x^2 - \log xy + 4\log y$

(b) $\ln(8x)^{\frac{1}{2}} + \ln 4x^2 - \ln(16x)^{\frac{1}{2}}$

(c) $e^6 e^{-6} = e^0 = 1$

7.

Find x if:

(a) $2\log_b 4 + \log_b 5 - \log_b 10 = \log_b x$

(b) $\log_b 30 - \log_b 5^2 = \log_b x$

(c) $\log_b 8 + \log_b x^2 = \log_b x$

(d) $\log_b(x+2) - \log_b 4 = \log_b 3x$

(e) $\log_b(x-1) + \log_b 3 = \log_b x$

8. If $\log_x\left(\frac{3}{4}\right) = -\frac{1}{3}$, find the value of x.

9. Simplify the following expressions.

a.) $3\log_4 2+4$

$\log_4 2^3 + 4$

$\log_4 8 + 4$

b.) $2\log_{16} 2$

$\log_{16} 4 = 1/2$

c.) $\left(\frac{-2x^2}{y^5}\right)^5$

$\frac{-32x^{10}}{y^{25}}$

d.) $\log_{\frac{1}{2}} 16$

$\left(\frac{1}{2}\right)^x = 16$

$x = -4$

e.) $\ln e^6 - \ln e^2$

$\ln \frac{e^6}{e^2} = \ln e^4 = 4$

f.) $\frac{1}{4}\log_6 81 - (2\log_6 6 - \frac{1}{2}\log_6 4)$

$\log_6 81^{1/4} - (\log_6 36 - \log_6 2)$

$\log_6 3 - (\log_6 18)$

$\log_6 \frac{3}{18}$

$\log_6 \frac{1}{6}$ $\quad -1$

10. The half-life of Iodine-131 is 8.1 days.. How long will it take a 50 gram sample to decay to 6.25 grams? Include an equation as part of your solution.

$y = a \cdot b^x$

$y = 50\left(\frac{1}{2}\right)^{x/8.1}$

$6.25 = 50\left(\frac{1}{2}\right)^{x/8.1}$

$\frac{6.25}{50} = \frac{1}{2}^{x/8.1}$

$\log_{.5} .125 = \frac{x}{8.1} \log_{.5} .5$

$3 = \frac{x}{8.1}$

$24.3 = x$

days

11. Solve the following equations. You may use any method as long as you show/describe your process.

a.) $2^{x-1} = \left(\frac{1}{8}\right)^{3x}$

$2^{x-1} = \left(2^{-3}\right)^{3x}$

$x - 1 = -9x$

$-1 = -10x$

$\frac{1}{10} = x$

b.) $\left(x^{\frac{-3}{5}} = 27\right)^{-5/3}$

$= \frac{1}{243}$

c.) $\log(x-3) = 2$

$10^2 = x - 3$

$103 = x$

d.) $2e^{x-3} - 5 = 12$

$2e^{x-3} = 17$

$e^{x-3} = 8.5$

$x - 3 = \ln 8.5$

$x = 5.14$

12. Marta has a savings account that pays 1.5% interest compounded monthly She has not made any deposits or withdrawals in many years. Suppose that her current balance is $4515.25. How much money was in her account 5 years ago? Include an equation as part of your solution.

$4515.25 = A\left(1 + \frac{.015}{12}\right)^{12(5)}$

$A = \$4189$

or $= 4515.25\left(1 + \frac{.015}{12}\right)^{12 \cdot (-5)}$

$= \$4189$

13. In 1980, a total of $119 trillion was spent on food and drinks in the United States. In 1994, the total was $274 trillion.

a.) Find an equation of the exponential function that can be used to model the total spent on food and drinks in the United States as a function of the number of years since 1980.

(0, 119)
(14, 274)

x → years after 1980
y → $ in trillions

$y = 119(b)^x$
$274 = 119(b)^{14}$
$2.303 = b^{14}$
$b = 1.06$
$y = 119(1.06)^x$

b.) Use your model to predict the amount spent in 1990? In 1975?

$t = 10$ 213 T
$t = -5$ 88.9 T

c.) Assuming this trend continues, estimate when the total sales will reach 500 trillion.

$500 = 119(1.06)^t$
24.6 ~2005
$4.2 = 1.06^t$

14. In 2010, the United States had the largest economy in the world 14.8 trillion, with China second at $9.7 trillion. The United States economy is predicted to grow at a continuous rate of 4.25%, while the Chinese economy will grow at 10.03%

a.) Which economy will be larger in the year 2020? Carefully detail your work/process.

US: $14.8e^{.0425t}$ $= 22.63$ T $t = 20$
C: $9.7e^{.103t}$ $= 27.1$ T

b.) Will the Chinese economy ever surpass the U.S. economy? Explain..

yes

15. According to an article in the New York Times (3/27/90), a wealthy Pennsylvania merchant named Jacob DeHaven loaned $450,000 to the Continental Congress in 1776 to rescue the troops at Valley Forge. The descendants of Mr. DeHaven sued the U.S. government for what they believed they were owed. The interest rate in effect in 1776 was 6% per year. How much did the family stand to collect in 1991? Assume that interest is compounded annually.

$y = 450000(1.06)^{2.5}$

$\approx 1.24 \times 10^{11}$

16. In 2006, Paul Cezanne's painting Still Life with Fruits and Ginger Jar sold at auction for $37 million. It's value grows by 12.6% annually. At this rate, how long will it take for the painting to double in value?

$y = 37(1.126)^x$

$74 = 37(1.126)^x$

$2 = 1.126^x$

$\ln 2 = x \ln 1.126$

$x \approx 5.84$ years

or

$2 = 1.126^x$

$\ln 2 = x$

5.94